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Data Driven Analytical Approach For Precise PV Module Annual Fixed Optimal Tilt Angle Determination

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***Abstract*— In this paper, data driven analytical approach for precise PV module annual fixed optimal tilt angle determination is presented. The focus in this work is to determine optimal annual fixed tilt angle using the annual average of the transposition coefficient. The approach required dataset of one-year solar radiation on the horizontal surface which is acquired for the case study site in Port Harcourt city in River State Nigeria with latitude and longitude of 4.78 and 7.01 respectively. The entire procedure requires two algorithms that are repeated two or more times before the precise optimal tilt angle can be obtained. The details of the two algorithms are presented along with requisite analytical models. The results of the first approximation of the optimal tilt angle obtained from the derivative of the trend line polynomial expression is 13.16552° whereas the optimal tilt angle obtained from the second implementation of the two algorithms is 15.55379747°. Notably, the maximum annual mean of transposition coefficient of 1.038023 is obtained at tilt angle of 15.5538 ° whereas the value of 1.03712 is obtained at tilt angle of 13.16552 ° which is about 0.1 % below the optimal value of 1.038023 obtained at tilt angle of 15.5538 °. The results obtained from the approach presented in this work show that the first approximation result of the optimal tilt angle gives about 99.9% of the transposition coefficient obtained with the optimal tilt angle. The approach went further to narrow down the tilt angle range that can give about 99.99 % of the optimal. So, when high precision optimal tilt angle is required, the procedure presented in this work is highly recommended.**

Keywords*— *Sun Path, PV Solar Power, Sun Elevation Angle, Annual Fixed Optimal Tilt Angle, Declination Angle

1. INTRODUCTION

Nowadays, solar power plants are increasingly being installed across the globe [1,2]. Due to the high investment cost of solar plants, efficiency of the plant is a key parameter used in the system design [3,4]. One of the factors that may affect the overall PV plant system energy yield is the PV module tilt angle [5,6]. When the PV module is optimally tilted, maximum solar radiation is incident on the PV plane and hence maximum energy is derived from the solar radiation. However, if the optimal tilt angle is violated, some energy will be lost due to misalignment of the PV module relative to the solar radiation direction.

Over the years, many analytical and empirical approaches have been used to estimate the optimal tilt angle for any given location across the globe [7,8]. However, when high precision optimal tilt angle is required more detailed procedure is needed. Accordingly, in this work, data driven analytical approach for precise PV module annual fixed optimal tilt angle determination is presented. The approach presented in this work utilizes daily solar radiation data for the horizontal plane and the sun declination and elevation angles to determine the solar radiation on tilted plane and the corresponding transposition coefficient. The transposition coefficient is then employed in a two stage algorithms to determine the optimal tilt angle that is more accurate than the conventional estimates of the optimal tilt angle.

2. METHODOLOGY

2.1 The analytical model for determination of solar radiation on tilted PV module

The optimal tilt angle, β_{opt} of PV module can be determine by computing $G_{\beta tilt}$ which is the solar radiation incident on the tilted PV module with tilt angle denoted as β_{tilt}. The value of $G_{\beta tilt}$ is computed from the knowledge of $G_{\beta=0}$ which is the solar radiation incident on the horizontal plain with $\beta_{tilt} = 0$. Specifically, $G_{\beta tilt}$ is given as [9];

$$G_{\beta tilt} = \frac{(G_{\beta=0})(Sin(\alpha_{Elev} + \beta_{tilt}))}{Sin(\alpha_{Elev})} \quad (1)$$

Where α_{Elev} is the sun elevation angle with respect to the PV module installation site latitude, denoted as ϕ, the declination angle denoted as δ_n and the hour angle denoted as ω_n, where [10,11];

$$\delta_n = 23.45 \text{ Sin}\left(\frac{360 \text{ x } (284 + \text{n})}{365}\right) \quad (2)$$

$$\omega_n = 15\,(TS - 12) \quad (3)$$

$$\alpha_{Elev} = \sin^{-1}[\text{Sin}\,(\delta_n)\text{Sin}\,(\varphi) + \text{Cos}\,(\delta_n)\text{Cos}\,(\varphi)\text{Cos}\,(\omega_n)] \quad (4)$$

Where n is the day number and TS is the solar time in hours. At solar noon TS = 12 and $\omega_n = 0°$. Then, the transposition coefficient, TC_β for any tilt angle is given as;

$$TC_\beta = \frac{G_{\beta tilt}}{G_{\beta=0}} = \frac{(G_{\beta=0})(Sin(\alpha_{Elev} + \beta_{tilt}))}{Sin(\alpha_{Elev})} \quad (5)$$

2.2 The procedure used to determine the optimal tilt angle based on the analytical model and solar radiation dataset

The approach used to determine the optimal tilt angle for annual fixed tilt angle is based on the daily solar radiation dataset. The dataset of one-year solar radiation on the horizontal surface is acquired for the case study site in Port Harcourt city in River State Nigeria. The site has latitude and longitude of 4.78 and 7.01 respectively. The scatter plot of the solar radiation data on the horizontal surface is presented in Figure 1. It has annual mean of 6.222712329 kW-hr/m^2/day.

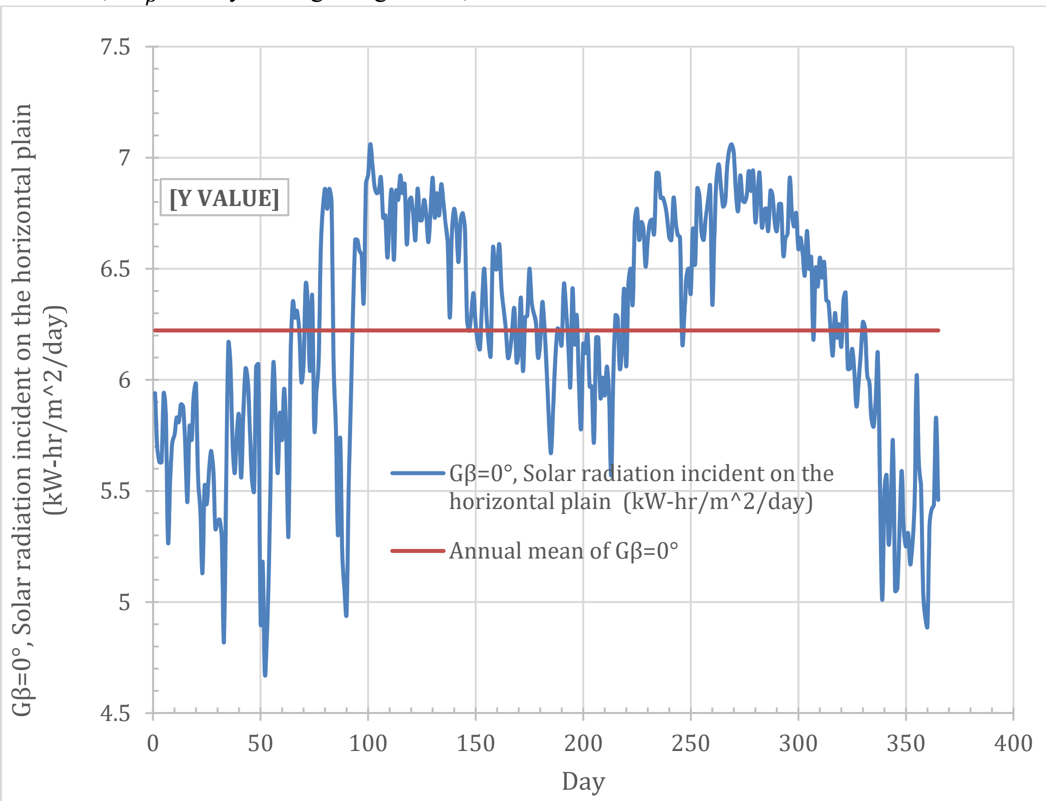

Figure 1 The scatter plot of the solar radiation data on the horizontal surface

The entire procedure requires two algorithms that may be repeated two or more times before the precise optimal tilt angle can be obtained. The first part of the procedure used to determine the precise optimal tilt angle is presented as Algorithm 1. In Algorithm 1 , the analytical models in Equations 1 to 5 are used along with the solar radiation dataset to compute the annual mean solar radiation on the tilted plane and the annual mean transposition factor for various tilt angles which is then used in Algorithm 2 to obtain the optimal tilt angle through trend line equation and solutions.

ALGORITHM 1:

Step 1: Obtain the one year daily dataset on $G_{\beta=0}$ which is the solar radiation data on the horizontal surface

Step 2: Initialize the tilt angle, counter k = 0°

Step 3: $\beta_{tilt(K)} = K$

Step 4: Initialize the sum, $SumG_{\beta tilt} = 0$

Step 5: Initialize the sum, $SumTC_{\beta(n)}$

Step 6: Initialize the day number, n =1

Step 7: Compute the declination angle, δ_n using Equation 2.

$$\delta_n = 23.45 \text{ Sin}\left(\frac{360 \text{ x } (284 + \text{n})}{365}\right)$$

Step 8: Compute the sun elevation angle α_{Elev} at solar noon with ω_n 0° using Equation 4.

$$\alpha_{Elev(n)} = \sin^{-1}[\text{Sin}\,(\delta_n)\text{Sin}\,(\varphi) + \text{Cos}\,(\delta_n)\text{Cos}\,(\varphi)\text{Cos}\,(\omega_n)]$$

Step 9: Compute the solar radiation incident on the tilted PV module, $G_{\beta tilt}$ using Equation 1.

$$G_{\beta tilt(n)} = \frac{\left(G_{\beta=0(n)}\right)\left(Sin\left(\alpha_{Elev(n)} + \beta_{tilt(k)}\right)\right)}{Sin\left(\alpha_{Elev(n)}\right)}$$

Step 10: Compute the transposition coefficient, $TC_{\beta(n)}$ using Equation 5.

$$TC_{\beta(n)} = \frac{G_{\beta tilt(n)}}{G_{\beta=0(n)}} = \frac{\left(G_{\beta=0(n)}\right)\left(Sin\left(\alpha_{Elev(n)} + \beta_{tilt(k)}\right)\right)}{Sin\left(\alpha_{Elev(n)}\right)}$$

Step 11: $SumG_{\beta tilt} = SumG_{\beta tilt} + G_{\beta tilt(n)}$

Step 12: $SumTC_{\beta(n)} = SumTC_{\beta(n)} + TC_{\beta(n)}$

Step 13: $AnnAvgG_{\beta tilt(k)} = \frac{SumG_{\beta tilt}}{n}$
Step 14: $AnnAvgSumTC_{\beta(k)} = \frac{SumTC_{\beta(n)}}{n}$
Step 15: Output $\beta_{tilt(k)}, G_{\beta tilt(n)}, TC_{\beta(n)}$
Step 16: n = n +1
Step 17: If n ≤ 365 Goto Step 7 ELSE Goto Step 18
Step 18: Output $\beta_{tilt(k)}, AnnAvgG_{\beta tilt(k)}$, $AnnAvgSumTC_{\beta(k)}$
Step 19: K = K + 5
Step 20: If K ≤ 90 Goto Step 3 ELSE Goto Step 21
Step 21: End

The output in Step 15 of Algorithm 1 is used to display the computed daily solar radiation on tilted PV module, $(G_{\beta tilt(n)})$ and the transposition coefficient $(TC_{\beta(n)})$ for that day for each of the tilt angles, $\beta_{tilt(k)}$. Such data can be used to determine the monthly, the seasonal and annual fixed optimal tilt angles.

On the other hand, the output in Step 19 of Algorithm 1 is used to display the annual average of the daily solar radiation on tilted PV module $(AnnAvgG_{\beta tilt(k)})$ and the annual average of the transposition coefficient $(AnnAvgSumTC_{\beta(k)})$ for each of the tilt angles. Such data is used to determine the annual fixed optimal tilt angle.

Furthermore, the annual fixed optimal tilt angle is the focus in this work and it is determined using (i) The annual average of the transposition coefficient, $AnnAvgSumTC_{\beta(k)}$ and

(ii) The annual average of the daily solar radiation on tilted PV module $AnnAvgG_{\beta tilt(k)}$.

Essentially, the two variables can used separately to determine the annual fixed optimal tilt angle and the results are validated using the one year daily dataset on $G_{\beta=0}$ and the algorithm presented in Algorithm 1. In this work, the annual average of the transposition coefficient is used to determine the optimal tilt angle.

2.2.1 Determination of the annual fixed optimal tilt angle using the annual average of the transposition coefficient

In order to determine the precise annual fixed optimal tilt angle using the annual average of the transposition coefficient, $AnnAvgSumTC_{\beta(k)}$, the following steps in Algorithm 2 are taken.

ALGORITHM 2.

Step 1: Use Algorithm 1 to generate table for $\beta_{tilt(K)}, AnnAvgSumTC_{\beta(k)}$ with tilt angle $\beta_{tilt(K)}$ ranging from k = 0° (in Step 2 of Algorithm 1) to k= 90° (in Step 20 of Algorithm 1) in increment of 5° (in Step 19 of Algorithm 1).

Step 2: Plot graph of $AnnAvgSumTC_{\beta(k)}$ versus $\beta_{tilt(K)}$ and fit in second order polynomial trend line equation on the graph where the trend line equation is of the form;

$$y = A(x^2) + B\,(x) + C \quad (6)$$

Where x in this case represent the tilt angle and y represent the solar radiation on the tilted PV module.

Step 3: Find the first derivative of the equation as follows:

$$\frac{\delta y}{\delta x} = 2(A)(x) + B \quad (7)$$

Step 4: Set the $\frac{\delta y}{\delta x} = 0$ and solve for x to get the optimal value.

$$x = \frac{B}{2(A)} \quad (8)$$

The value of X obtained in this instance with $\beta_{tilt(K)}$ ranging from k = 0° to k= 90° may not be the exact value of the optimal tilt angle, as can be seen or verified from the graph. However, it will enable the determination of the first approximation of the optimal tilt angle.

This first approximation is used to determine the narrow range of values to be used for fine-tuning the optimal tilt angle. In this case, the value of $\beta_{tilt(K)}$ ranging from k = maximum (0, X-5) to k= minimum (90, X +5) is used and the increment for K is 0.5. With this settings, Algorithm 1 is run followed by Algorithm 2. At this second run of Algorithm 1 and Algorithm 2, the value of optimal tilt angle obtained will be more accurate. The procedure in Algorithm 2 can be repeated again to improve on the accuracy of the optimal tilt angle obtained.

3. RESULTS AND DISCUSSIONS

The results of the declination angle for the 365 days in year is shown in Figure 2 while the results of the elevation angle at solar noon for the 365 days in year is shown in Figure 3. Again, the daily solar radiation on the horizontal (βtilt = 0 °) and on the tilted PV module with βtilt = 60 ° are shown in Figure 4. Similar graph of the daily solar radiation on the tilted PV module with βtilt = 60 ° are shown in Figure 5. Similar graph of the daily solar radiation on the tilted PV module with βtilt = 10 ° are shown in Figure 5. The results in Figure 4 and Figure 5 show that with βtilt = 10 ° the annual mean daily solar radiation on the tilted PV module is 6.42047271 kW-hr/m^2/day which is above the value of 6.222712329 kW-hr/m^2/day for horizontal plan with βtilt = 0 °. However, with βtilt = 60 ° the annual mean daily solar radiation on the tilted PV module is 4.569113715 kW-hr/m^2/day which is below the value of 6.222712329 kW-hr/m^2/day for horizontal plan with βtilt = 0 °. Essentially, PV tilt angle of 10 ° better that PV tilt angle of 60 ° which is worse than PV tilt angle of 0 °. In any case, the optimal tilt angle is expected to give the highest value of daily solar radiation on the tilted PV module.

Also, the annual mean transposition coefficient with βtilt = 60 ° and average of 0.741078503 is shown in Figure 6 while the annual mean transposition coefficient with βtilt = 10 ° and average of 1.0331468 is shown in Figure 7. The results again show that tilt angle of 10 ° is better as it is about 103.3 % of the solar radiation that would have been captured if the PV tilt angle is zero (that is if the PV module is flat on the horizontal plane) whereas the value obtained with tilt angle of 60 ° is about 74.1% of the solar radiation that would have been captured if the PV tilt angle is zero. At this point, the optimal tilt angle is required to optimize the solar radiation incident on the tilted PV module.

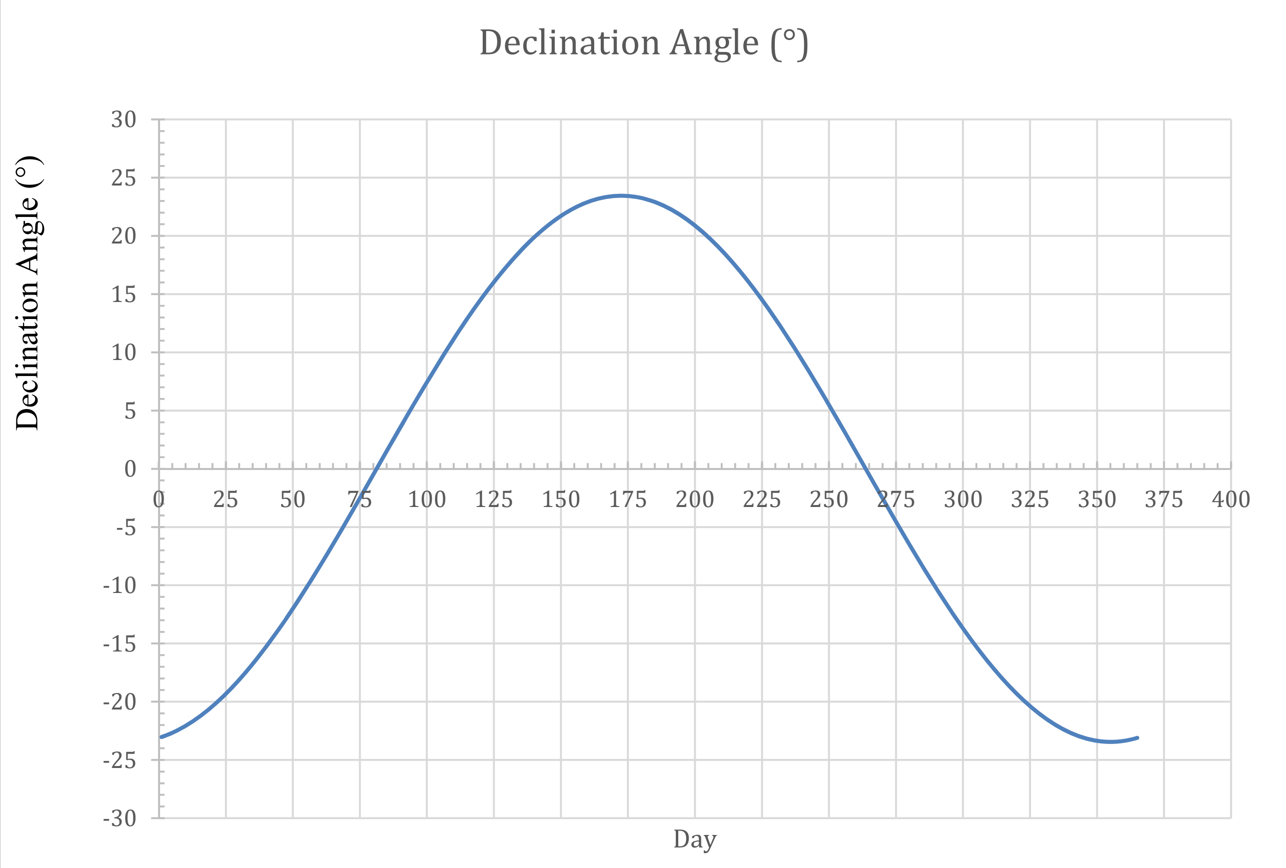

Figure 2 The declination angle for the 365 days

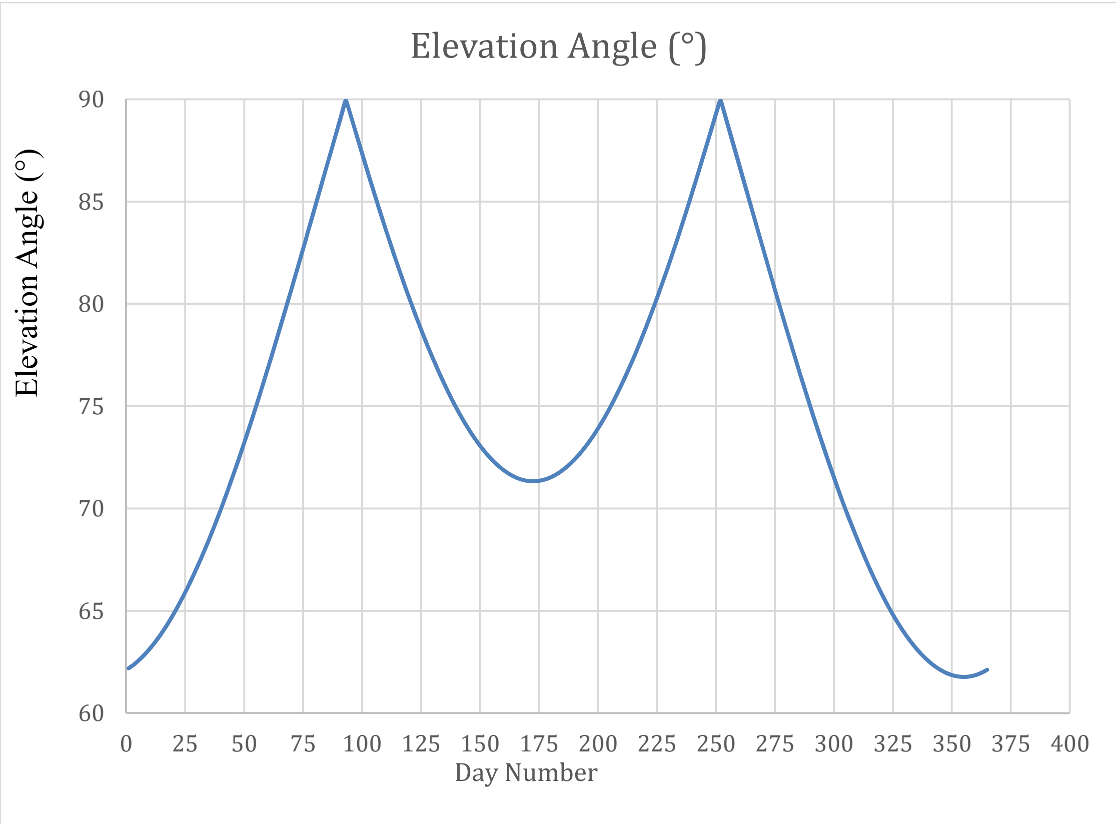

Figure 3 The elevation angle at solar noon for the 365 days and geo-coordinate with latitude and longitude of 4.78 and 7.01 respectively

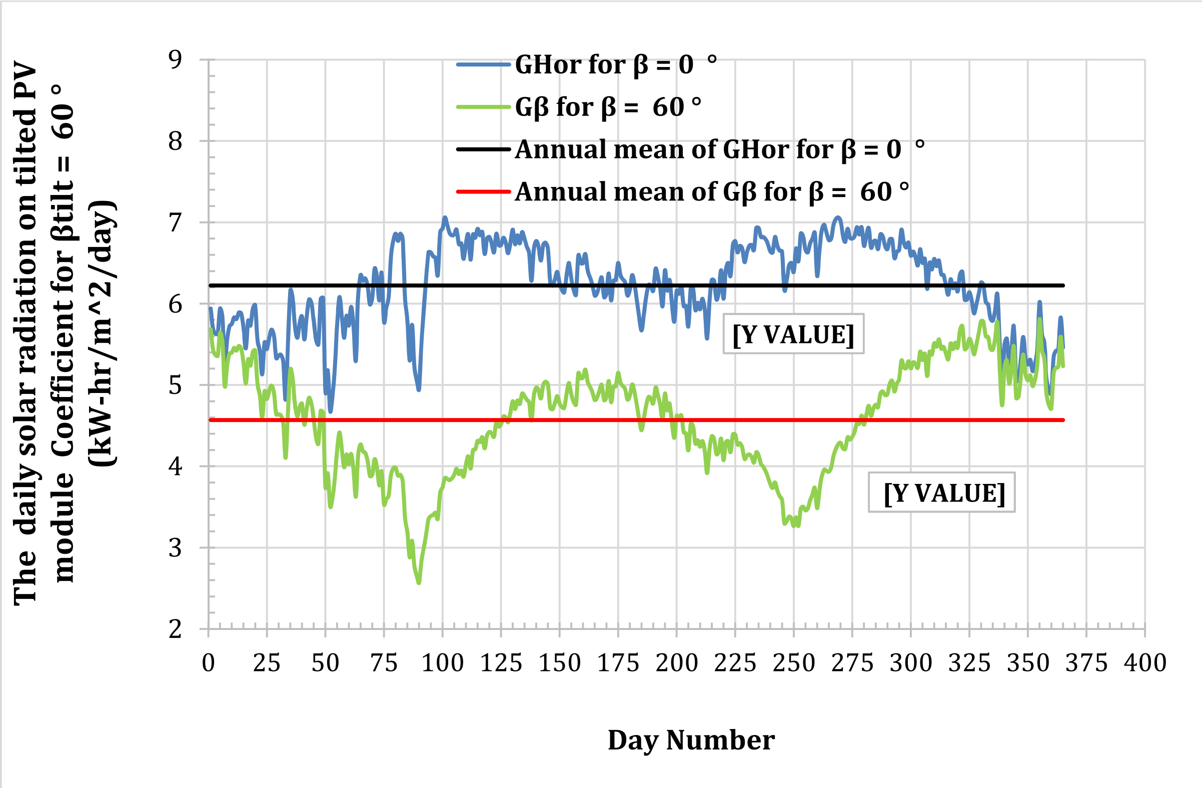

Figure 4 The daily solar radiation on the horizontal (βtilt = 0 °) and on tilted PV module with βtilt = 60 °

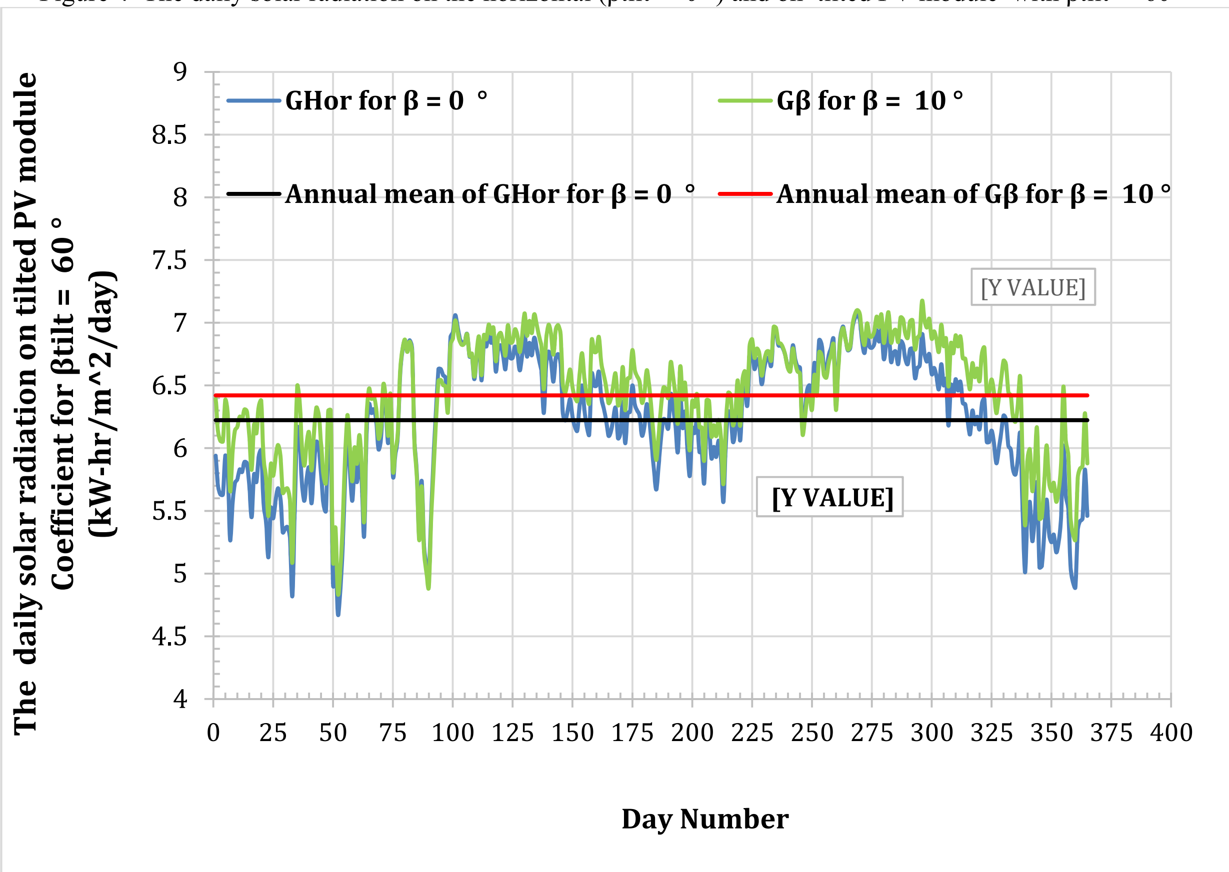

Figure 5 The daily solar radiation on the horizontal (βtilt = 0 °) and on tilted PV module with βtilt = 10 °

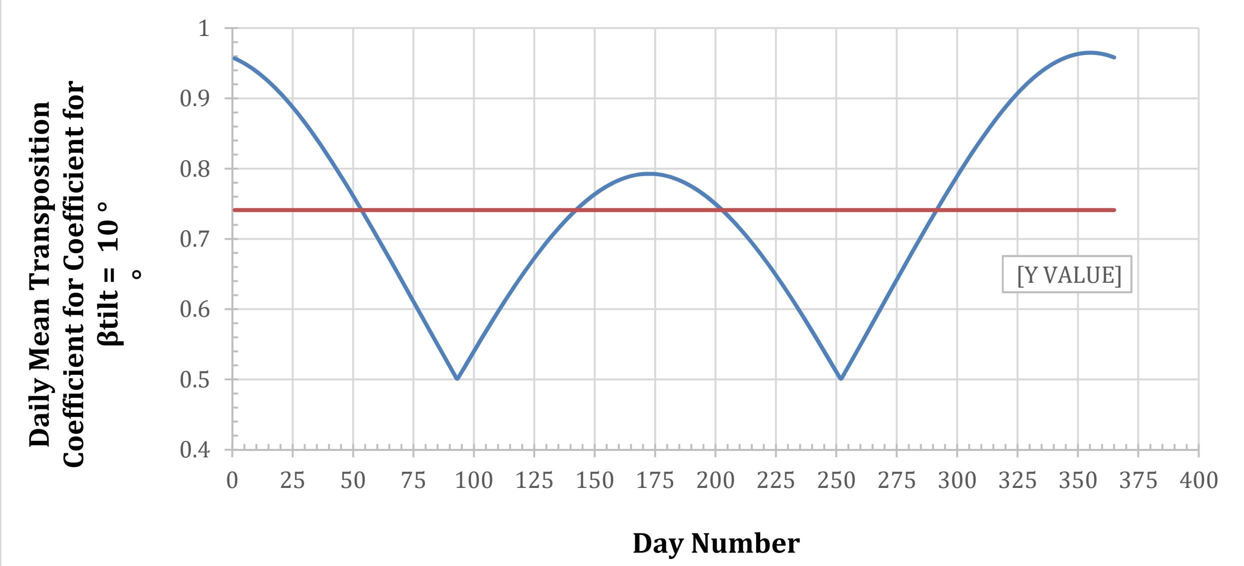

Figure 6 The daily mean transposition coefficient for βtilt = 60 °

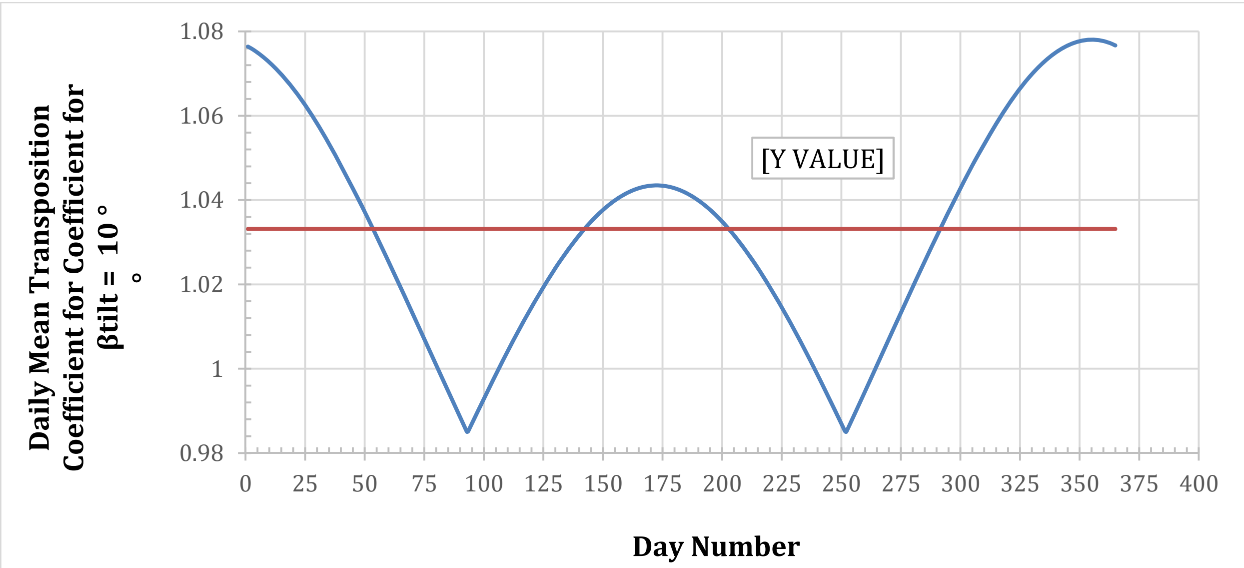

Figure 7 The daily transposition coefficient for βtilt = 10 °

The results of the annual mean solar radiation and annual mean transposition coefficient for tilt angle $\beta_{tilt(K)}$ ranging from k = 0° to k= 90° in increment of 5° are shown in Table 1 while the graph of annual mean transposition coefficient versus tilt angle $\beta_{tilt(K)}$ ranging from k = 0° to k= 90° in increment of 5° is shown in Figure 8. From the graph in Figure 8, the optimal tilt angle obtained from the derivative of the trend line polynomial expression in Equation 10 is 13.16552°.

Based on the procedure in Algorithm 2, the tilt angle range is narrowed down to the values of 10° to 19°. The results of the annual mean solar radiation and annual mean transposition coefficient for tilt angle $\beta_{tilt(K)}$ ranging from k = 10° to k= 19° in increment of 0.5° are shown in Table 2 while the graph of annual mean transposition coefficient versus tilt angle $\beta_{tilt(K)}$ ranging from k = 10° to k= 19° in increment of 0.5° is shown in Figure 9. From the graph in Figure 9, the optimal tilt angle obtained from the derivative of the trend line polynomial expression in Equation 15.55379747°.

The results of the comparison of the annual mean of transposition coefficient for βtilt = 13.16552 ° and for βtilt = 15.5538° are shown in Table 3, Figure 10 and Figure 11. The results in Table 3 and Figure 10 showed that the maximum annual mean of transposition coefficient of 1.038023 is obtained at tilt angle βtilt = 15.5538 ° whereas the annual mean of transposition coefficient of 1.03712 is obtained at tilt angle βtilt = 13.16552 ° which is about 0.1 % below the optimal value of 1.038023 obtained at tilt angle of 15.5538 °, as shown in Figure 11. In addition, from the results in Figure 11, it can be stated that tilt angle between 13.0 ° and 18.2° gives at least 99.9 % of the optimal tilt value of 1.038023 obtained at tilt angle of 15.5538 ° while tilt angle between 14.75 ° and 16.25° gives at least 99.99 % of the optimal tilt value. So, when high precision optimal tilt angle is required, the procedure presented in this work is highly recommended.

Table 1 The annual mean solar radiation and annual mean transposition coefficient for tilt angle $\beta_{tilt(K)}$ ranging from k = 0° to k= 90° in increment of 5°

Tilt angle, β (°)	Annual Mean of GHor for β = 0 °	Annual Mean of Gβ for β = 1 °	Annual Mean of Transposition Coefficient for β = 1 °
1	6.222712329	6.251141752	1.004705982
4.78	6.222712329	6.341336952	1.019718869
5	6.222712329	6.345739976	1.020456545
10	6.222712329	6.42047271	1.0331468
15	6.222712329	6.446341769	1.037974184
20	6.222712329	6.423150276	1.034901957
25	6.222712329	6.351074731	1.023953502
30	6.222712329	6.230663672	1.005212143
35	6.222712329	6.0628335	0.978820512
40	6.222712329	5.848861504	0.944979465
45	6.222712329	5.590376141	0.903946555
50	6.222712329	5.28934464	0.856034065
55	6.222712329	4.948058032	0.80160664
60	6.222712329	4.569113715	0.741078503
65	6.222712329	4.155395684	0.674910312
66	6.222712329	4.068751918	0.661043469
67	6.222712329	3.980868771	0.646975266
70	6.222712329	3.710052582	0.603605646
75	6.222712329	3.236473741	0.527707176
80	6.222712329	2.738263379	0.447792537
85	6.222712329	2.21921318	0.364469925
90	6.222712329	1.683273429	0.278373478

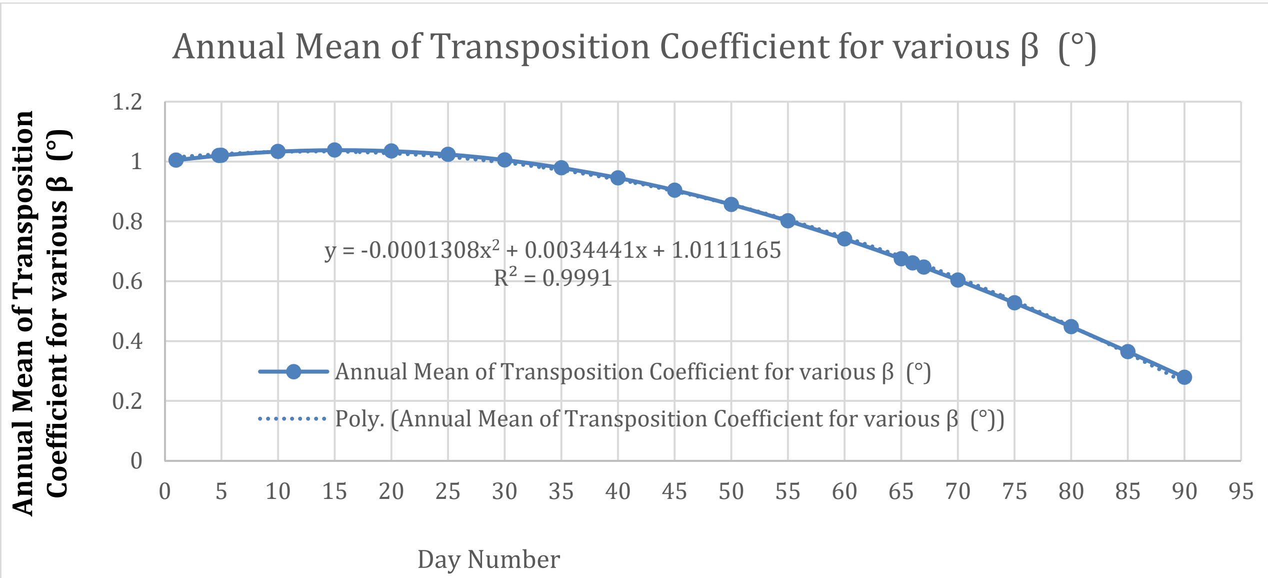

Figure 8 The Annual Mean transposition coefficient for various values of tilt angle, β ranging from 0° to 90 °

$$Y = -0.0001308(X)^2 + 0.0034441(X) + 1.0111165 \quad (9)$$

$$\frac{\delta Y}{\delta X} = -0.0002616(X) + 0.0034441 \quad (10)$$

Therefore, X= 13.16552°

Table 2 The annual mean solar radiation and annual mean transposition coefficient for tilt angle $\beta_{tilt(K)}$ ranging from k = 10° to k= 19 in increment of 0.5°

Tilt angle, β (°)	Annual Mean of GHor for β = 0 °	Annual Mean of Gβ for β = 10 °	Annual Mean of Transposition Coefficient for β = 10 °
10	6.222712329	6.42047271	1.0331468
10.5	6.222712329	6.425264653	1.033984445
11	6.222712329	6.429567289	1.034743347
11.5	6.222712329	6.433380287	1.03542345
12	6.222712329	6.436703359	1.036024701
12.5	6.222712329	6.439536251	1.036547055
13	6.222712329	6.441878747	1.036990472
13.5	6.222712329	6.44373067	1.037354918
14	6.222712329	6.445091877	1.037640366
14.5	6.222712329	6.445962266	1.037846793
15	6.222712329	6.446341769	1.037974184
15.5	6.222712329	6.446230359	1.038022529
16	6.222712329	6.445628044	1.037991825
16.5	6.222712329	6.444534868	1.037882074
17	6.222712329	6.442950917	1.037693284
17.5	6.222712329	6.44087631	1.03742547
18	6.222712329	6.438311205	1.037078652
18.5	6.222712329	6.435255798	1.036652856
19	6.222712329	6.431710321	1.036148115

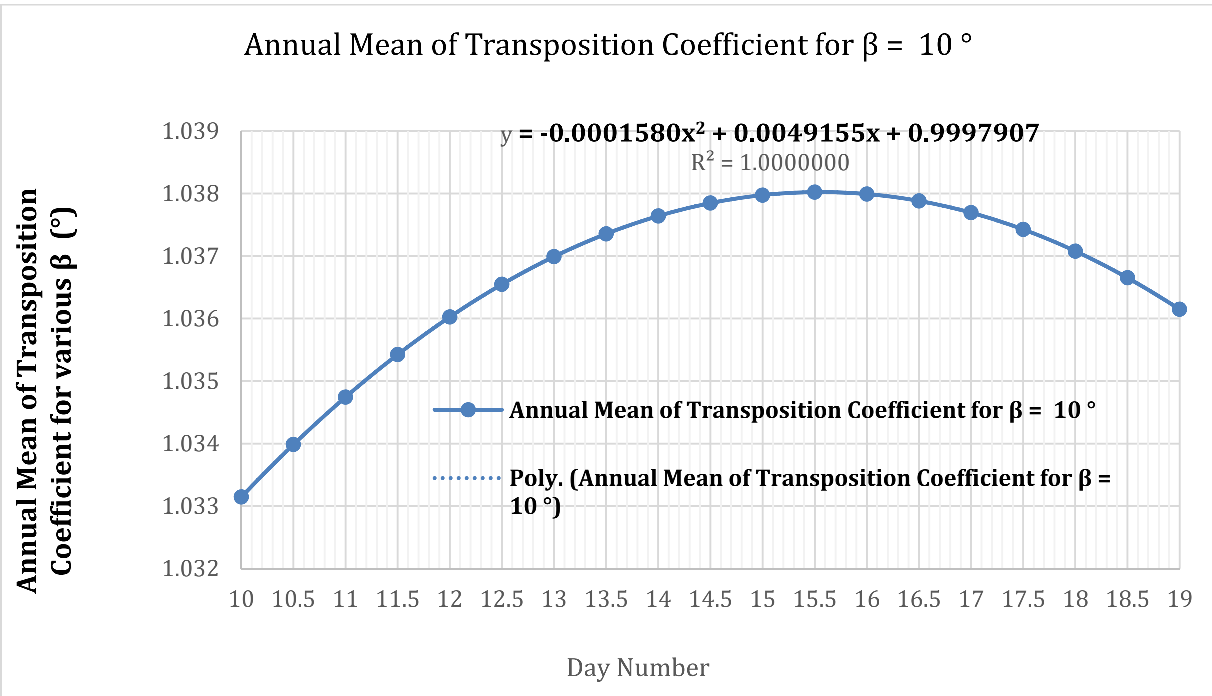

Figure 9 The Annual Mean transposition coefficient for various values of tilt angle, β ranging from 10° to 19 °

$$Y = -0.0001580(X)^2 + 0.0049155(X) + 0.9997907 \quad (11)$$

$$\frac{\delta Y}{\delta X} = -0.000316\,(X) + 0.0049155 \quad (12)$$

Therefore,

X= 15.55379747°

Table 3 Comparison of the annual mean of transposition coefficient for βtilt = 13.16552 °and for βtilt = 15.5538°

Tilt angle, β (°)	Annual Mean of Transposition Coefficient for βtilt = 10 °	Percentage difference below the optimal value (%)
13	1.03699	0.10
13.16552	1.03712	0.09
13.5	1.03735	0.06
14	1.03764	0.04
14.5	1.03785	0.02
15	1.03797	0.00
15.5	1.03802	0.00
15.5538	1.038023	0.00
16	1.03799	0.00
17	1.03769	0.03

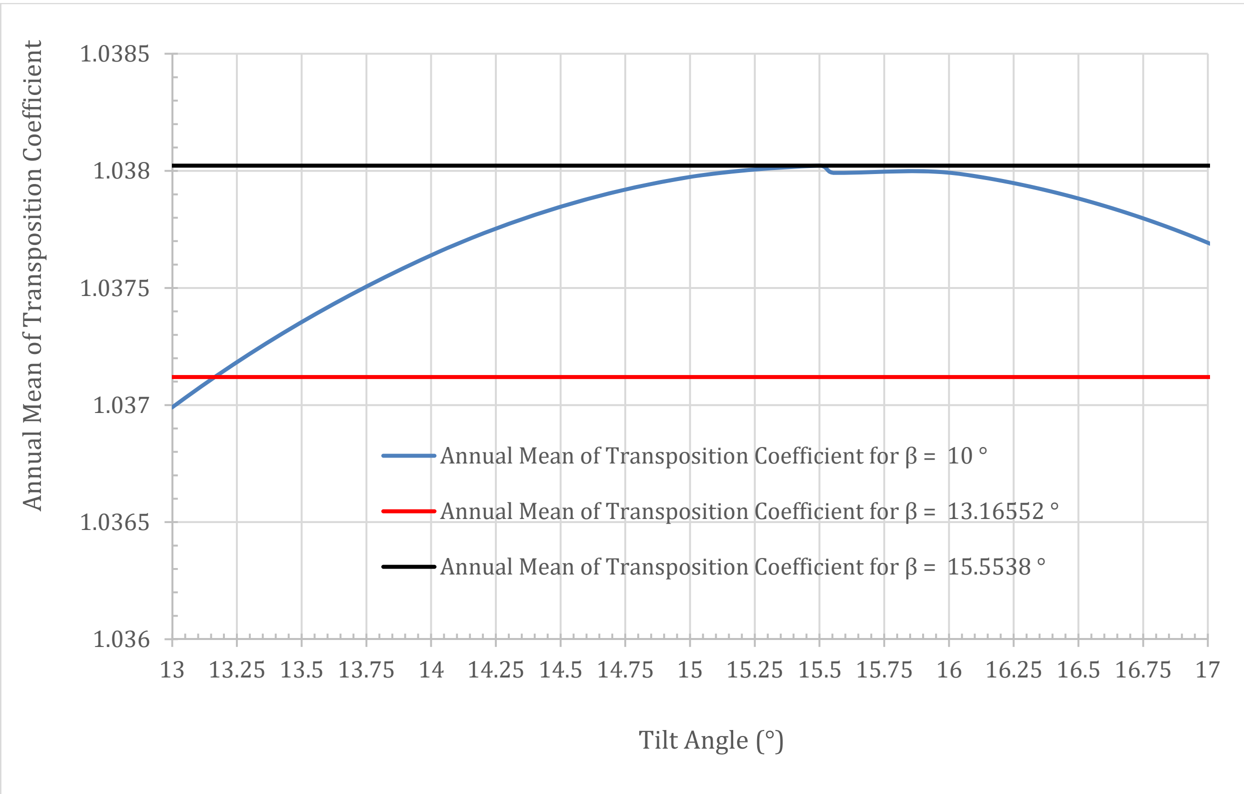

Figure 10 Comparison of the annual mean of transposition coefficient for βtilt = 13.16552 °and for βtilt = 15.5538°

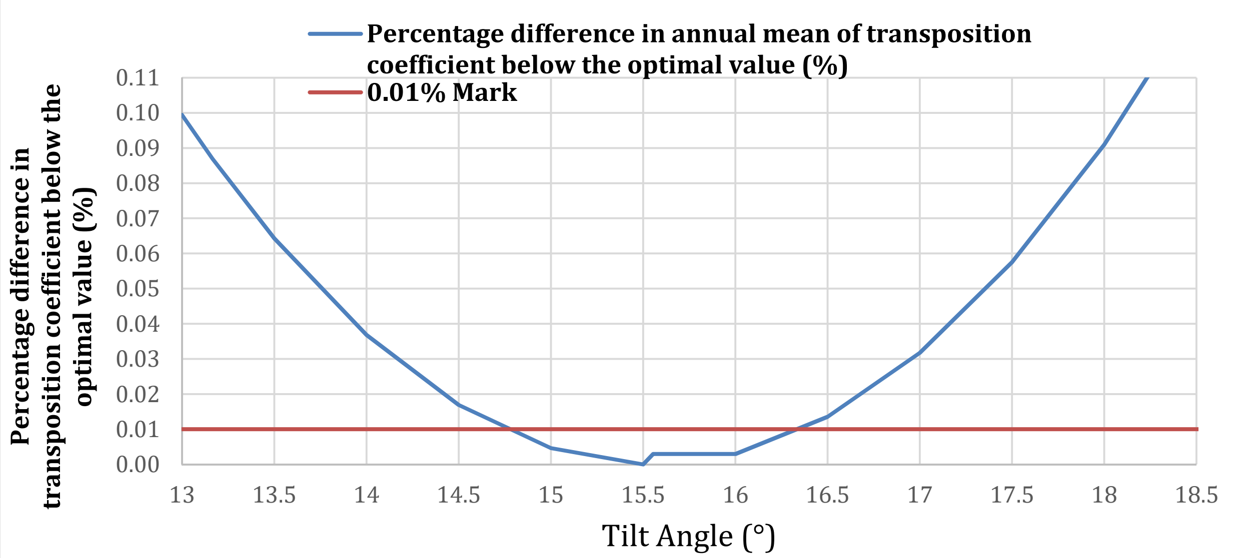

Figure 11 Percentage difference in annual mean of transposition coefficient below the optimal value at βtilt = 15.5538°

4. CONCLUSION

An approach to obtain accurate optimal tilt angle for solar photovoltaic power module installation is presented. The approach utilized analytical model and solar radiation data of the installation site to determine the annual mean transposition coefficient which is then used to determine the optimal tilt angle. Due to error in the trend line model equation used in characterizing the annual mean transposition coefficient as function of the tilt angle, the approach adopted a two stage procedure whereby the trend line model is used first on the tilt angle that has wider range to locate the first approximation of the optimal tilt angle. Then, the tilt angle range is narrowed down to values very close to the first approximation of the optimal tilt angle. The results obtained from the approach presented in this work show that the first approximation result of the optimal tilt angle gives about 99.9% of the transposition coefficient obtained with the optimal tilt angle. The approach went further to narrow down the tilt angle range that can give about 99.99 % of the optimal.

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